

Practical implementation of semi-active control through response prediction algorithms

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ABSTRACT

Research over the past several decades into semi-active control has provided useful insight into the practical applications of this type of control on actual structures. Hardware and software limitations, though, have made the implementation of such control very difficult. Time delays in both hardware and software make practical implementation differ greatly from that of theory. These delays, together with a prediction algorithm that may help overcome some of these, are presented in the current paper. The effectiveness of the prediction algorithm is tested analytically and experimentally using sine and random response waves.

1 INTRODUCTION

Many new and innovative hardware and software solutions have brought the prospect of widespread semi-active control in structures ever closer. Sophisticated algorithms together with new semi-active devices promise to significantly reduce structural response.

Several articles on semi-active control algorithms and approaches (Inaudi & Hayen 1995, Occhiuzzi & Serino 1995) seem very promising. Theoretical work has shown great reductions in the response of structures under earthquake or wind loading. Experimental work with these and other algorithms is, though, very limited.

A main problem with the implementation of such algorithms, and thus such control regimes, are the hardware and software delays between sensing the structural motion and applying control. In the control algorithm proposed by Inaudi & Hayen (1995) one must know when the structure reaches a peak displacement response so that a hydraulic damper becomes effective in dissipating energy: this is a requirement for maximum energy dissipation and thus optimum control. The control algorithm by Serino & Occhiuzzi (1996) has a similar requirement in that one must know exactly when structural velocity has opposite sign of base velocity so that control may be applied.

Hardware and software limitations in terms of both computer acquisition and actuation of control devices make it imperative that some form of structural response prediction is used to overcome the control delay problem. Minimum delay times for conventional acquisition systems are in the range of 10-20 ms. This is a considerable amount of time for structural semi-active control applications which directly corresponds to a loss in the response reduction capacity and energy dissipation characteristics of a control regime. If the mechanical delay times of the control devices, which are currently in the range of 10-40 ms, are added to this then it is easy to see the necessity of response prediction.

2 HARDWARE AND SOFTWARE DELAYS

As reported in Serino & Russo (1997), a prototype semi-active oleodynamic damper, designed by Serino and manufactured by FIP INDUSTRIALE, has been tested extensively using the SISTEM (Seismic ISolator TEst Machine) testing apparatus, owned by ENEA and located at the Structural Dynamic Testing Laboratory of ISMES in Seriate (Bergamo, Italy). The DG XII of the European Union within the framework of the Training and Mobility of Researchers (TMR) Program, Action 2, financed the experimental campaign: Access to Large-Scale Facilities. Through the tests, the authors were able to pinpoint the response delays of the damper. It was found that mechanical opening valve delays (time interval between start of control impulse to the electro-valve and beginning of oil flow) were around 20 ms while mechanical closing valve delays (time interval between end of control pulse and stop of oil flow) varied from 15 to 75 ms

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depending on pulse duration. These delays, which can be attributed to solenoid coil and hydraulic fluid reaction, are representative of this type of device.

Additional delays were found to be related to the inevitable software delays that typical acquisition and control programs have. The program used during the tests for acquisition and control was LabVIEW 5.0 (National Instruments 1998a,b). It was found that the analog to digital conversion, computer computation according to the selected control algorithm and digital to analog conversion to apply the required command pulse were responsible for approximately 10 ms further delay in device response. The programs containing the control algorithms were reviewed by National Instruments of Italy (LabVIEW suppliers) who found that the above software delay times could not be brought down any further. Again, these delay times are typical of programs of this nature.

It should therefore be clear now that the above mentioned delay times depend on the device and software in use. If these are known, it may be possible to start the control pulse sufficiently in advance with respect to a response peak so that the electro-valve will actually open at the correct time as prescribed by the control algorithm in use. In the following paragraphs, two simple response prediction methods are presented. They are both based on the assumption that the response (e.g. displacement) of a generic instrumented point of the controlled structure can be divided in a finite number of segments, each corresponding to a half period sine wave characterized by a certain amplitude and frequency.

3 4-POINT "SINE-FIT" PREDICTION METHOD

3.1 Derivation of prediction formulas

The first method needs four equidistant (in time) acquired points at the beginning of the generic sine wave segment, which is detected by the change in sign of the second derivative of the response time history. The method allows to predict the amplitude and frequency of the half period sine wave which best fits the actual response segment, as well as its phase difference and vertical shift from the horizontal time axis.

Assuming that the acquired response quantity is a displacement and that the readings are continuously taken at a pre-set time interval Δt , the beginning of the generic sine wave segment corresponds to the change in sign of acceleration $\ddot{x}(t)$,

computed using the well known central difference expression:

$$\ddot{x}_{i-1} = \frac{x_i - 2x_{i-1} + x_{i-2}}{\Delta t^2} \quad (1)$$

where $x_i = x(t)$, $x_{i-1} = x(t - \Delta t)$ and $x_{i-2} = x(t - 2\Delta t)$ are three acquired points in sequence. Fitting four subsequently acquired displacements to a sine wave we have:

$$x_0 = A \cdot \sin(\omega t_0 - \varphi) + B \quad (2)$$

$$x_1 = A \cdot \sin(\omega t_1 - \varphi) + B, \quad t_1 = t_0 + \Delta t \quad (3)$$

$$x_2 = A \cdot \sin(\omega t_2 - \varphi) + B, \quad t_2 = t_1 + \Delta t \quad (4)$$

$$x_3 = A \cdot \sin(\omega t_3 - \varphi) + B, \quad t_3 = t_2 + \Delta t \quad (5)$$

where A = sine wave amplitude, ω = circular frequency, φ = phase difference and B = vertical shift, are four unknown quantities to be determined. Deriving twice with respect to time and employing the central difference expression (1) above,

we get: $\ddot{x}_1 = -A \omega^2 \cdot \sin(\omega t_1 - \varphi) = \frac{x_2 - 2x_1 + x_0}{\Delta t^2}$ and $\ddot{x}_2 = -A \omega^2 \cdot \sin(\omega t_2 - \varphi) = \frac{x_3 - 2x_2 + x_1}{\Delta t^2}$ (6)&(7)

Substitution of (3) and (4) into (6) and (7) gives:

$$-\omega^2(x_1 - B) = \frac{x_2 - 2x_1 + x_0}{\Delta t^2} \quad \text{and} \quad -\omega^2(x_2 - B) = \frac{x_3 - 2x_2 + x_1}{\Delta t^2} \quad (8) \& (9)$$

which can be seen as a system of two equations in the two unknowns B and ω . Its resolution yields:

$$\omega = \frac{1}{\Delta t} \cdot \sqrt{3 - \frac{x_3 - x_0}{x_2 - x_1}} \quad \text{and} \quad B = \frac{x_3 x_1 - x_2 x_0 - x_2^2 + x_1^2}{x_3 - 3x_2 + 3x_1 - x_0} \quad (10) \& (11)$$

From equation (10) and (11) one can see that the requirements for the acquired points are:

$$a. \quad x_2 - x_1 \neq 0 \quad \Rightarrow \quad \dot{x}_1^a = \dot{x}_2^b = \frac{x_2 - x_1}{\Delta t} \neq 0$$

$$\text{b.} \quad 3 - \frac{x_3 - x_0}{x_2 - x_1} > 0 \Rightarrow \ddot{x}_1^f = \ddot{x}_2^b = \frac{x_3 - 3x_2 + 3x_1 - x_0}{2\Delta t^3} < 0, \text{ if } x_2 > x_1$$

$$\ddot{x}_1^f = \ddot{x}_2^b = \frac{x_3 - 3x_2 + 3x_1 - x_0}{2\Delta t^3} > 0, \text{ if } x_2 < x_1$$

where the superscript *f* and *b* denote forward and backward finite differences, respectively. The above requirements are always met when the four displacement values are acquired in the first half of the half period sine wave, where the velocity $\dot{x}(t)$ is different from zero and the jerk $\ddot{x}(t)$ is negative for increasing values of $x(t)$ and positive when $x(t)$ is decreasing.

To obtain the other two unknowns *A* and φ , we now derive once with respect to time and use the central difference expression for the first derivative:

$$\dot{x}_1 = A \omega \cdot \cos(\omega t_1 - \varphi) = \frac{x_2 - x_0}{2\Delta t}, \quad \dot{x}_2 = A \omega \cdot \cos(\omega t_2 - \varphi) = \frac{x_3 - x_1}{2\Delta t} \quad (12), (13)$$

Dividing (13) by (12) and taking into account that:

$$\cos(\omega t_2 - \varphi) = \cos[(\omega t_1 - \varphi) + \omega \Delta t] = \cos(\omega t_1 - \varphi) \cdot \cos(\omega \Delta t) - \sin(\omega t_1 - \varphi) \cdot \sin(\omega \Delta t) \quad (14a)$$

we get:

$$\cos(\omega \Delta t) - \tan(\omega t_1 - \varphi) \cdot \sin(\omega \Delta t) = \frac{x_3 - x_1}{x_2 - x_0} \quad (15)$$

and thus, solving for φ :

$$\varphi = \omega t_1 - \text{atan} \left[\frac{x_3 - x_1}{(x_0 - x_2) \cdot \sin(\omega \Delta t)} + \frac{1}{\tan(\omega \Delta t)} \right] \quad (16a)$$

Alternatively, using the relation:

$$\cos(\omega t_1 - \varphi) = \cos[(\omega t_2 - \varphi) - \omega \Delta t] = \cos(\omega t_2 - \varphi) \cdot \cos(\omega \Delta t) + \sin(\omega t_2 - \varphi) \cdot \sin(\omega \Delta t) \quad (14b)$$

Dividing (12) by (13) another solution for φ is found:

$$\varphi = \omega t_2 - \text{atan} \left[\frac{x_2 - x_0}{(x_3 - x_1) \cdot \sin(\omega \Delta t)} - \frac{1}{\tan(\omega \Delta t)} \right] \quad (16b)$$

Note that it also always is:

$$\text{c.} \quad x_0 - x_2 \neq 0 \quad \text{and} \quad x_3 - x_1 \neq 0$$

when the four displacement values are acquired in the first half of the half-period sine wave.

$$\text{Finally, } A \text{ is obtained from equation (12) or (13):} \quad A = \frac{x_2 - x_0}{2\omega \Delta t \cdot \cos(\omega t_1 - \varphi)}, \quad A = \frac{x_3 - x_1}{2\omega \Delta t \cdot \cos(\omega t_2 - \varphi)} \quad (17a), (17b)$$

Expressions (10), (11), (16) and (17) form the basis of the 4-point "sine-fit" prediction method, while a., b. and c. are the basic requirements to obtain real finite values of the unknowns. As can be seen from above, all that is required for the prediction are four acquired points at the beginning of any wave segment and the sampling time interval Δt . From numerous theoretical and experimental tests the algorithm has proven to be robust as long as adequate DAQ resolution is used. As resolution drops so does the accuracy of the prediction.

3.2 4-Point acquisition resolution

To determine the resolution needs of the 4-point "sine fit" prediction method, several extreme acquisition error cases were examined. Figure 1 shows the case that gives, for a given constant positive (negative) Δx representing the resolution error, the maximum (minimum) value of ω , see equation (10):

$$\omega' = \frac{1}{\Delta t} \cdot \sqrt{3 - \frac{(x_3 - \Delta x) - (x_0 + \Delta x)}{(x_2 + \Delta x) - (x_1 - \Delta x)}}, \quad \text{Solving for } \Delta x \text{ one derives: } \Delta x = \frac{(x_3 - x_0) - (x_2 - x_1)(3 - \omega'^2 \Delta t^2)}{2 \cdot (4 - \omega'^2 \Delta t^2)} \quad (18), (19)$$

Assuming $\omega' = \alpha \cdot \omega$, the above formula provides the maximum $\Delta x > 0$ (minimum $\Delta x < 0$) for a given allowable deviation from ω represented by an $\alpha > 1$ ($\alpha < 1$) value. To understand how resolution can be defined, let us consider four subsequently acquired displacement values at sampling time interval Δt :

$$x_i = A \cdot \sin \omega t_i' + B, \quad i = 0, 1, 2, 3 \quad (20)$$

with $0 < t'_i = t_i - \bar{t} < \pi/2\omega$, where \bar{t} is the time at which the latest change in sign of acceleration occurred, so that the requirements a., b. and c. above are satisfied. Therefore, if the DAQ system has a range (in Volt) of exactly $2A$, from equation (19) we obtain minimum required acquisition resolution:

$$\frac{\Delta x}{2A} = \frac{(\sin\alpha t'_3 - \sin\alpha t'_0) - (\sin\alpha t'_2 - \sin\alpha t'_1)(3 - \omega^2 \Delta t^2)}{4 \cdot (4 - \omega^2 \Delta t^2)} \quad (21)$$

In other terms, considering that:

$$\sin \omega t'_i = \sin \left(t'_{12} + \frac{2i-3}{2} \Delta t \right) = \sin \omega t'_{12} \cdot \cos \left(\frac{2i-3}{2} \omega \Delta t \right) + \cos \omega t'_{12} \cdot \sin \left(\frac{2i-3}{2} \omega \Delta t \right) \quad (22)$$

where $t'_{12} = (t'_1 + t'_2)/2 = t'_1 + \Delta t/2$, and assuming $t'_{12} = n\Delta t$ we get:

$$\frac{\Delta x}{2A} = \cos(n\omega\Delta t) \cdot \frac{\sin\left(\frac{3}{2}\omega\Delta t\right) - \sin\left(\frac{\omega\Delta t}{2}\right)(3 - \alpha^2(\omega\Delta t)^2)}{2 \cdot (4 - \alpha^2(\omega\Delta t)^2)} \quad (23)$$

which can be seen as a function of n , $\omega\Delta t$ and α only. When this is translated into DAQ board resolution $B = [\text{bit}]$, we

have:
$$\frac{1}{2^B} = \frac{\Delta x}{2A} = f(n, \omega\Delta t, \alpha), \quad \text{that is: } B = -\log_2 f(n, \omega\Delta t, \alpha) \quad (24), (25)$$

In Figure 2 the above expression is plotted for $\alpha = 0.8 \div 1.2$ and some typical values of n and $\omega\Delta t$. For example, if $n = 10$, $\omega = 20.00 \text{ rad/s}$ and $\Delta t = 0.00125 \text{ s}$ ($\omega\Delta t = 0.025$), a DAQ board of 24-bit allows to limit error on ω within $0.968 < \alpha < 1.032$. Therefore, resolution requirements for the 4-point "sine fit" method are realistic as 24-bit resolution DAQ boards are in existence. Though, as this type of board is still fairly expensive and not yet widely used in engineering, a second method with similar characteristics but less dependant on resolution is proposed.

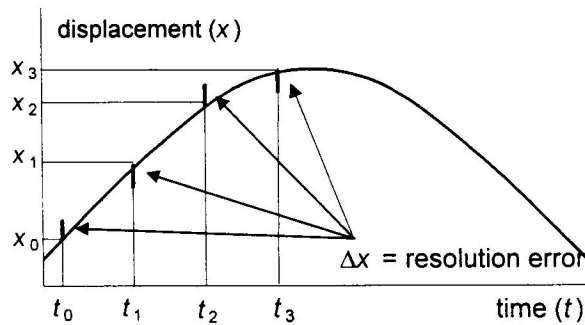


Figure 1. Extreme acquisition error case (4-point sine-fit).

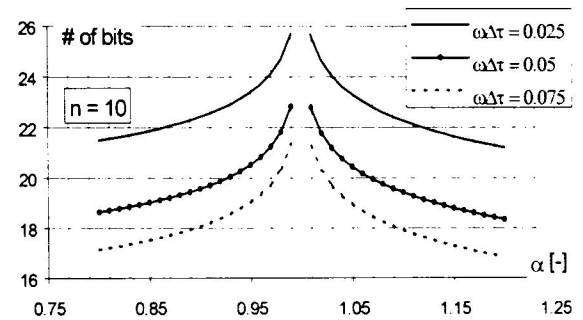


Figure 2. DAQ board resolution requirements (4-point sine-fit).

4 3-POINT "SINE-FIT" PREDICTION METHOD

4.1 Derivation of prediction formulas

The 3-point prediction method is a specific case of the 4-point one: when vertical axial transition is negligible, it is convenient to assume $B = 0$ in the "sine-fit" expressions above, with the advantage that it is necessary to acquire only three points to make the prediction instead of four. Assuming again a sampling time interval Δt , let us consider now three displacement values acquired in sequence immediately after a change in sign of acceleration:

$$x_1 = A \cdot \sin(\omega t_1 - \varphi) \quad (26)$$

$$x_2 = A \cdot \sin(\omega t_2 - \varphi) \quad t_2 = t_1 + \Delta t \quad (27)$$

$$x_3 = A \cdot \sin(\omega t_3 - \varphi) \quad t_3 = t_2 + \Delta t \quad (28)$$

Note that the unknown quantities to be determined are now three (A , ω and φ). Deriving (27) twice with respect to time and adopting central difference, we get

$$\ddot{x}_2 = -A \omega^2 \cdot \sin(\omega t_2 - \varphi) = \frac{x_3 - 2x_2 + x_1}{\Delta t^2} \quad (29)$$

Thus from (27) and (29): $-\omega^2 x_2 = \frac{x_3 - 2x_2 + x_1}{\Delta t^2}$, which solved for ω gives: $\omega = \frac{1}{\Delta t} \cdot \sqrt{\frac{x_3 - 2x_2 + x_1}{x_2}}$ (30),(31)

Considering that: $\dot{x}_2 = A \omega \cdot \cos(\omega t_2 - \varphi) = \frac{x_3 - x_1}{2\Delta t}$, dividing (27) by (32) we obtain: $\tan(\omega t_2 - \varphi) = \frac{2\omega \Delta t \cdot x_2}{x_3 - x_1}$ (32), (33)

and thus, solving for φ : $\varphi = \omega t_2 - \text{atan}\left(\frac{2\omega \Delta t \cdot x_2}{x_3 - x_1}\right)$ (34)

Finally, A is derived from (26), (27) or (28): $A = \frac{x_i}{\sin(\omega t_i - \varphi)}$ $i = 1, 2, 3$ (35)

Expressions (31), (34) and (35) summarize the 3-point prediction method, while the requirements to obtain real finite values of the unknowns are now:

- $x_2 \neq 0$
- $-\frac{x_3 - 2x_2 + x_1}{x_2} > 0 \Rightarrow \begin{cases} \ddot{x}_2 < 0, & \text{if } x_2 > 0 \\ \ddot{x}_2 > 0, & \text{if } x_2 < 0 \end{cases}$
- $x_3 - x_1 \neq 0 \Rightarrow \dot{x}_2 = \frac{x_3 - x_1}{2\Delta t} \neq 0$

always met when the displacements are acquired in the first half of the half-period sine wave.

4.2 3-Point acquisition resolution

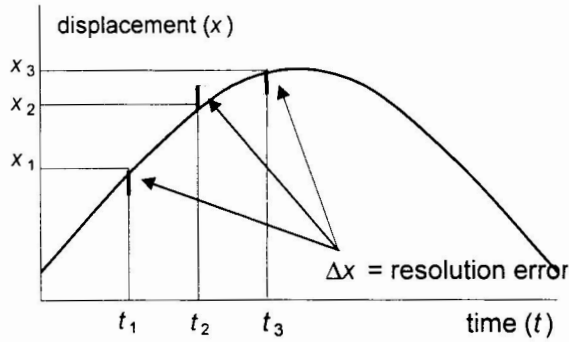


Figure 3. Extreme acquisition error case (3-point sine-fit).

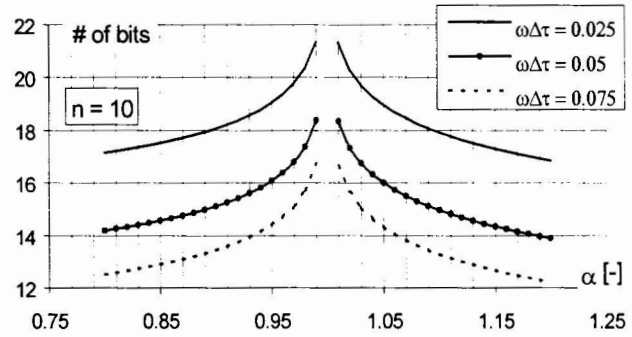


Figure 4. DAQ board resolution requirements (3-point sine-fit).

Figure 3 shows the case that gives, for a given constant positive (negative) resolution error Δx , the maximum (minimum) value of ω , see equation (31):

$$\omega' = \frac{1}{\Delta t} \cdot \sqrt{2 - \frac{(x_3 - \Delta x) + (x_1 - \Delta x)}{x_2 + \Delta x}}, \quad \text{Solving for } \Delta x \text{ one derives: } \Delta x = \frac{x_1 + x_3 - x_2(2 - \omega'^2 \Delta t^2)}{4 - \omega'^2 \Delta t^2} \quad (36) (37)$$

To define acquisition resolution, we assume as above $\omega' = \alpha \cdot \omega$ and consider three subsequently acquired displacement values at sampling time Δt :

$$x_i = A \cdot \sin \omega t'_i \quad i = 1, 2, 3 \quad (38)$$

with $0 < t'_i = t_i - \bar{t} < \pi / 2\omega$. Dividing both terms of equation (37) by the supposed DAQ system range $2A$, considering that:

$$\sin \omega t'_i = \sin [\omega t'_2 + (i-2)\Delta t] = \sin \omega t'_2 \cdot \cos [\omega(i-2)\Delta t] + \cos \omega t'_2 \cdot \sin [\omega(i-2)\Delta t] \quad (39)$$

and assuming $t'_{12} = n\Delta t$, we obtain minimum required acquisition resolution:

$$\frac{\Delta x}{2A} = \sin(n\omega \Delta t) \cdot \frac{2 \cos \omega \Delta t - 2 + \alpha^2 (\omega \Delta t)^2}{4 - \alpha^2 (\omega \Delta t)^2}, \quad \text{which is again a function of } n, \omega \Delta t \text{ and } \alpha \text{ only.} \quad (40)$$

In Figure 4 DAQ board resolution $B = [\text{bit}]$ is plotted for $\alpha = 0.8 \div 1.2$ and some typical values of n and $\omega \Delta t$. For

example, if $n = 10$ and $\omega\Delta t = 0.05$, a DAQ board of 16-bit allows to limit error on ω within $0.947 < \alpha < 1.049$. As can be seen, the resolution requirements needed, for the 3-point method, are relatively small compared to that offered by modern DAQ hardware (most DAQ boards now work with at least 16-bit resolution).

5 VERIFICATION TESTS AND CONCLUSIONS

A one story frame structure equipped with semi-active oleodynamic dampers was extensively tested on the ISMES shaking table using both harmonic and earthquake excitation. The control algorithm was operating so that an opening signal was given to the electro-valves when structural displacement response reached a relative maximum or minimum.

In Figure 5 the displacement and electro-valve signals recorded during harmonic excitation tests when the control program was operating without prediction are plotted superimposed: approximately 10 ms delays are observed between start of electro-valve signals and displacement peaks. Figure 6 shows the same signals when prediction was used with a 20 ms advance on the opening valve command: the ability of the prediction method to anticipate electro-valve opening is evident.

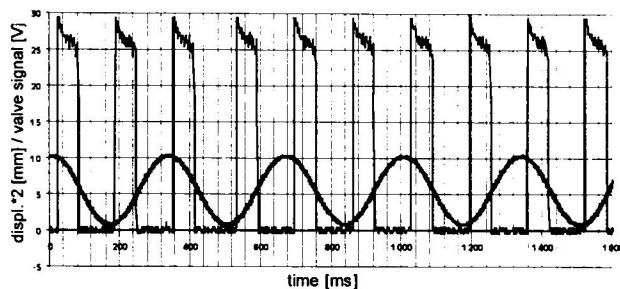


Figure 5. Harmonic excitation tests: no prediction.

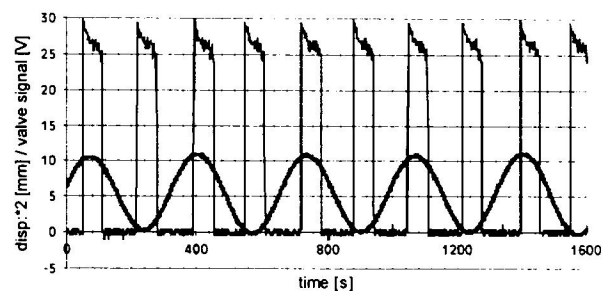


Figure 6. Harmonic tests: prediction with 20 ms advance.

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